

SIMULATION AND ASSIMILATION OF SATELLITE ALTIMETER DATA AT THE OCEANIC  
MESOSCALE

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## ABSTRACT

An improved Objective Analysis technique (Gandia, 1963) is used along with an altimeter signal statistical model, an altimeter noise statistical model, an orbital model, and synoptic surface current maps in the POLYMODE-SDE area, to evaluate the performance of various observational strategies in catching the mesoscale variability at mid-latitudes. In particular, simulated repetitive nominal orbits of ERS-1, TOPEX, and SPOT/POSEIDON are examined. Our statistical models are consistent with previous in-situ and remote-sensed results (Fu, 1983). Our own results show the critical importance of the existence of a subcycle, scanning in either direction. Moreover, long repeat cycles (>20 days) and short cross-track distances (<300 km) seem preferable, since they match mesoscale statistics. Another goal of our study is to prepare and discuss Sea-Surface Height assimilation in quasigeostrophic models. Our restored SSH maps are shown to meet that purpose, if an efficient extrapolation method or deep in-situ data (floats) are used on the vertical to start and update the model.

## 1. INTRODUCTION

Active and passive satellite-flown sensors have been extensively shown to open new perspectives in geophysical sciences; in particular, the altimeters of GEOS 3 and SEASAT are helping us in the study of the dynamics of mesoscale ocean currents. New products are now available, such as wavenumber spectra (Fu, 1983; Menard, 1983) or global maps of the statistics of the variability (Douglas et al., 1983; Cheney et al., 1983), and sample synoptic maps (Robinson et al., 1983) of which oceanographers have to take advantage. Three further altimeter missions which are currently planned include the European ERS-1, the American TOPEX, and the French SPOT (POSEIDON experiment). There are many ways to evaluate the altimeter missions of these satellites, and many parameters to optimize.

The direct measurement of sea surface height is an achievement of great scientific importance and the coverage provided by a satellite data base is unique in its space-time extent and along-track density. However, now and in the conceivable future, such data will be available only during a very few costly special missions. Moreover, a shortcoming of remotely sensed data to the deep sea oceanographer is the fact that only surface variables are measured. Thus every effort must be made to plan sampling strategies and analysis schemes to optimize the scientific utility of the observations. We are engaged in research to address several relevant questions along these lines. Our approach involves simulating the acquisition and analysis of altimetric data sets by performing operations on four-dimensional oceanic or oceanic-like fields. These fields are obtained by either i) combining existing real ocean data obtained from various in situ instruments or ii) using existing observations to drive a numerical ocean model (Robinson, 1984). In the first case we fly a simulated altimeter over historical data and in the second case we fly a simulated altimeter over simulated data. In our analysis we include the quantitative

investigation of the construction of deep ocean fields combined from both remotely sensed and in situ observations. Finally we mention our research on the assimilation of such observed fields in dynamical ocean models (Robinson & Leslie, 1984; Robinson et al., 1984) for initialization and verification purposes and for dynamical interpolations and optimal estimates.

Here we describe preliminary experiments; firstly of observational network evaluation, i.e. optimization of sampling, in order to improve mesoscale mapping and statistics (Section 3), and secondly the construction of current maps (Sections 2 and 3) for the assimilation of the sea-surface height information in our quasigeostrophic model (Section 4).

## 2. SATELLITE ALTIMETER SIMULATIONS IN THE POLYMODE AREA

In order to sample Sea Surface Height (SSH) data as a satellite would do, an orbital model is needed. The requirements on this model are not very severe, since we only want to infer statistical conclusions on the coverage, and get synoptic mid-ocean maps of SSH. A simple circular orbit model has been used, along with accurate orbital parameters of future missions (ERS-1, TOPEX, SPOT). No atmospheric drag is involved. The repeat cycle, cross-track distance, inclination angle, and subcycles are adequately modelled. The numerical clock is accurate to 1 sec for more than 1 year, in order to ensure exact repetitivity during long simulations. The altitude calculated by the model is accurate to 1%.

Linear optimal estimation is used to restore SSH maps. This is an 'intelligent' interpolation method, whose estimator is statistically data-adaptive. It allows the integration of other remote-sensed and in situ measurements, and is consistent with the data assimilation techniques currently in use at Harvard (e.g. Tu, 1981; Robinson, 1983). The estimation method is a space-time extension of the Gandin (1963) Objective Analysis method. Phase propagation is built into the signal correlation model. A limited set of influential points is selected at each interpolation point (Carter, 1983). An improved algorithm eliminates data points which are more correlated to each other than to the interpolation point. The need for such a stringent selection arises from the one-dimensional nature of the data.

Restored SSH maps can be used for operational system assessment, which is inferred from error estimates provided by Objective Analysis (see Section 3) or by initializing and updating the Harvard quasi-geostrophic baroclinic model (see Section 4).

We have used two different data sets to represent sea-truth. The first is a 1000 km simulation generated by the Harvard model using regionally tuned statistics in the POLYMODE-SDE region. The second sea-truth we have chosen is the 100 dbar level of the POLYMODE-SDE Mark II data set produced at Harvard. An interpolation scheme, similar to the one described above, has been used to combine POLYMODE-SDE Soviet current-meter moorings (1400-700 dbar) and XBTs (400-100 dbar), from Julian day 3345 (July 21, 1977) through Julian day 3725 (August 5, 1978). A mean salinity profile has been used to compute shallow geostrophic velocities relative to the 700 dbar absolute velocity level. The grid has 281.25 km long sides and is centered on 29.00 N, 70.00 W.

SSH is sampled from the sea-truth along arcs given by the orbital model. The streamfunction in the shallow level is interpreted in terms of pressure, which is converted into SSH by  $f_0/g$ . An overall mean is subtracted from the SSH time series

in the domain. Individual passes are assumed to be perfectly unbiased and detrended. As an illustration of the SSH variability in the whole POLYMODE-SDE Mark II data set, the following statistics are derived:

Mean elevation = 1.41 m ( ref. to 700 dbar)  
 Variance = 860 cm<sup>2</sup>  
 Standard deviation = 29.33 cm  
 Minimum (rel. to mean) = -28 cm  
 Maximum (rel. to mean) = +33 cm  
 Max. amplitude = 61 cm  
 Data count (XBTs) = 1336

The variance above includes SSH variability at time scales much larger than the typical mesoscale time. Thus, the standard deviation order of magnitude of 10 cm has to be kept in mind.

A gaussian noise of 3 cm standard deviation is added to SSH time series.

The SSH correlations in the Mark II data set exhibit the following scales:

Space: SW-NE direction: 110 km                      Time: e-folding: 10-20 days  
           SE-NW direction: 160 km

The space correlation scale is defined as the zero-crossing of the covariances.

In Section 3, an isotropic stationary signal correlation model has been used. It has the following analytical expression (see Fig. 1):

$$C(r,t) = (1 + ar - \frac{1}{3} a^3 r^3) e^{- (ar + t^2/R_{ct}^2)} \quad (1)$$

with:

$$\begin{aligned} a &= 2.1038/R_{cx} \\ R_{cx} &= \text{correlation scale} \\ R_{ct} &= \text{correlation time} \\ r &= ((x-c_x t)^2 + (y-c_y t)^2)^{1/2} . \end{aligned}$$

Using this expression leads to a -2 power law for the kinetic energy spectrum. The following values have been used:

$$\begin{aligned} R_{cx} &= 150 \text{ km} \\ R_{ct} &= 13 \text{ days for 1000 km runs (error only)} \\ &\quad 20 \text{ days for 281.25 km runs (with sea truth)} \\ c_x &= 0 \\ c_y &= 0 . \end{aligned}$$

Residual SSH power spectra, as obtained by Menard (1983) or Fu (1983), along repeat tracks of SEASAT altimeter data, can be used to calculate the signal correlation. If the along-track wavenumber spectrum is denoted by  $E_1(k_1)$ , the one-dimensional isotropic correlation function is written:

$$C(r) = \int_{-\infty}^{+\infty} E_1(k_1) \exp(2\pi i k_1 r) dk_1, \quad (2)$$

i.e. the Fourier transform of the spectrum. A typical correlation function for the mid-latitude south Atlantic is given on Fig. 2. For wavelengths shorter than 300 km,

the shape of this function shows good agreement with the shape of (1).

Clearly, the Objective Analysis technique cannot be more accurate than the statistics used to build the estimator. The question of confidence levels on variance and covariances is thus of critical importance. The availability of large data sets of SSH sampled on a homogeneous mesh on large space and time scales is a decisive advantage of satellites compared to in situ sensors: the statistics derived from remotely sensed data are likely to be much more reliable. In the POLYMODE-SDE area, estimates based upon GEOS 3 data (Douglas et al., 1983) and SEASAT data (Cheney et al., 1983) are available. The 3.5 year calculation of SSH variability from GEOS 3 (Fig. 3) yields a typical value of 10 cm. The 24 day calculation from SEASAT (Fig. 4) gives 5 cm. The discrepancy is due to considerable attenuation of the mesoscale signal (50-150 days) in the second case. The first value of 10 cm is more reliable, and in good agreement with the calculations from in situ measurements discussed above.

The noise level, denoted by E, has been set to 20% of the SSH variance. The residual noise in the altimeter data, using the repeat track method, is expected to be 5 cm at the mesoscale, or less, for SEASAT and future missions:

- Residual tropospheric error: 1-2 cm/50-500 km  
(Rain and water vapor)
- Atmospheric loading: 3 cm/200-1000 km  
(Inverse barometer effects)
- Residual geoid: a few centimeters/all scales  
(Tracking errors)

Assuming the 10 cm variability of Douglas et al., we find that  $E = 25\%$ , which is likely to be pessimistic if the data have been correctly undersampled along passes.

### 3. TWO EXAMPLES: ERS-1 and SPOT

Figs 5 and 6 are RMS expected error maps for two 1000 km simulations carried out with regionally tuned statistics in the POLYMODE-SDE area, resp. with ERS-I and SPOT. The following orbital parameters are used:

- |          |                                   |
|----------|-----------------------------------|
| ERS-1:   | $14 + \frac{1}{3}$ rotation/day   |
| (Fig. 7) | repeat cycle: 3 days              |
|          | cross-track distance: 910 km      |
|          | inclination angle: $98.52^\circ$  |
| SPOT:    | $14 + \frac{5}{26}$ rotations/day |
| (Fig. 8) | repeat cycle: 26 days             |
|          | cross-track distance: 106 km      |
|          | inclination angle: $98.72^\circ$  |

These two satellites stand for the two possible extremes as far as regional mid-latitude coverage is concerned. TOPEX (Fig. 9) is just in between (repeat cycle: 10 days). ERS-1 and SPOT are sun-synchronous: the angle between ascending and descending passes (respectively  $24.85^\circ$  and  $25.33^\circ$  at the equator) is sharper than for TOPEX ( $44.61^\circ$  at the equator).

Let us define the overbar as the space and time average on the maps on an

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integer number of repeat cycles (6 for ERS-1 = 18 days, 1 for SPOT = 26 days),  $\sigma^2(x,y,t)$  as the local expected error in terms of signal variance,  $v$  as the signal variance,  $h(x,y,t)$  and  $\hat{h}(x,y,t)$  as the actual and restored SSH. The following statistics are derived for a 1000 km domain:

	ERS-1	SPOT
Global RMS expected error = $\frac{\sigma^2}{2}$ <sup>1/2</sup> :	71.90%	56.15% of st. dev.
Global MS expected error = $\frac{\sigma^2}{2}$ :	51.70%	31.50% of var.
Global average expected error (bias):	0	0

ERS-1 exhibits an excellent localized resolution, and a high variance of error, due to the absence of a subcycle to scan between adjacent tracks. The inhomogeneity in resolution would probably result in improper model assimilation.

SPOT shows a good general resolution, and a low error variance. The minimum error (29%) is higher, but the global statistics are much better. The whole error field propagates with the subcycle, i.e. at 25.1 cm/s, westward. Scanning between two successive tracks, with a subcycle of 5 days, is responsible for the good behavior of SPOT. TOPEX has a subcycle of 3 days, scanning eastward at 122.2 cm/s. SPOT is probably better for longer periods and relatively slow-moving events, while TOPEX is likely to perform better for energetic, rapidly evolving events at the advective time scale. Besides, TOPEX is not sunsynchronous, thus releasing the constraint of an almost polar orbit. The angle between ascending and descending passes is more open, and the chances to catch a mesoscale event are higher.

Fig. 10 shows a 5-day sequence of SSH simulation and reconstruction, in the 281.25 km domain sampled by SPOT. Also shown are the actual error field and the expected error field, both expressed in signal variance units. The following overall statistics are derived from the 26-day simulation:

		ERS-1
Mark-II domain		
Global RMS expected error = $\frac{\sigma^2}{2}$ <sup>1/2</sup> :		43.11% of st. dev.
Global MS expected error = $\frac{\sigma^2}{2}$ :		18.58% of variance
Global average expected error :		0
Global RMS actual error = $\frac{(h-\hat{h})^2}{2}$ <sup>1/2</sup> / $v$ <sup>1/2</sup> :		40.18% of st. dev.
Global MS actual error = $\frac{(h-\hat{h})^2}{2}$ / $v$ :		16.14% of variance
Global average actual error = $\frac{h-\hat{h}}{2}$ / $v$ <sup>1/2</sup> :		4.95% of st. dev.

The bias is clearly negligible. It is striking to see how close the expected and actual values are. Objective Analysis can thus be expected to be a good method to interpolate along-track data, and the altimeter to catch accurately the variance, even in this small domain.

The introduction of phase speeds in the correlations, and the use of a larger domain are both likely to lead to more definitive conclusions on the different sampling schemes.

#### 4. ASSIMILATING SEA-SURFACE HEIGHTS IN QG MODELS

Considerable recent progress has been made in assimilating data into the Harvard QG open ocean model (Robinson and Leslie, 1984; Robinson et al., 1984). Both barotropic and baroclinic experiments have been carried out in order to resolve questions as to the source(s) of accuracy and error in both the objective analysis and the dynamical forecast (for details of the model, see: Tu, 1981; Miller et al., 1983). Figs. 11a-c show a barotropic experiment involving the assimilation of sea surface heights in the QG model. Fig. 11a is the model generated 'Veritas' data set at period 3.0. The objective analysis is performed on the 'Veritas' data sampled with the MODE-1 observational network (Fig. 11b) and with the MODE-1 network and a simulated satellite track 'cross' (Fig. 11c). The addition of the satellite track restores much of the missing structure of the 'Veritas' data. These SSH maps are used to initialize the model, which computes the evolution of streamfunction using the discretized QG equations. There is good agreement between sea-truth and the model outputs, and the westward propagation is restored.

Fig. 11d illustrates the results of a 17 day baroclinic forecast using real data. Note that forecast in this 144 km domain experiment maintains (with the exception of some spikes) a level of rms difference between forecast and data of less than 25%. The spikes have been attributed to 'unrepresentative' hydrocasts in the data set. This forecast experiment does not involve use of remotely sensed data or a satellite simulation. This remains as a next step for the POLYMODE-SDE region. The Harvard model has been initialized with Rossby waves, simulated data, simulated satellite observed sea-surface heights (barotropic), and real data and run over flat bottoms and real topography. 'This model is proving to be an efficient and accurate component in the prediction and description of fields in open ocean regions of various internal dynamics' (Robinson and Leslie, 1984).

In the summer of 1983 a significant step in data assimilation was accomplished by carrying out a real time forecast in the regime of turbulent jets and eddies in the California Current (Robinson et al. 1984). The dynamical model successfully predicted the appearance of a zonal jet in the center of the experimental region and also provided the means of identifying a major eddy-eddy interaction event. The implications of these results for the effective exploitation of satellite altimetric data for practical forecasts and scientific studies are substantial.

#### 5. CONCLUSION

The analyses and simulations presented here show the ability of objective analysis to map along-track altimeter data in a suitable way for dynamical model initialization, and to evaluate sampling strategies. A set of global statistical parameters have been defined for that purpose. The two case studies on ERS-1 and SPOT show the great interest of the subcycle, which, as a matter of fact, doubles the capabilities of the instrument, by being tuned on two time scales (5 days and 26 days). Furthermore, it can be shown (Fig. 12) that the most economical way to set two tracks apart is to equal their distance to the horizontal correlation scale. This is also true in time with the repeat cycle and the correlation time. 5 days and 26 days stand approximately for the advective and linear time scales at mid-latitudes respectively. The SPOT observational strategy is thus regionally adapted, as far as sampling parameters are concerned. It is still unclear whether in situ data, along with altimeter data, are compulsory for assimilation. Different techniques of extrapolation on the vertical are currently tried in Harvard, in order to initialize and update properly the dynamical model for multi-level runs. The efficient use of these

techniques and of those described above can be considered an essential step towards our knowledge of the oceanic mesoscale on large space and time scales.

## 6. ACKNOWLEDGEMENTS

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## FIGURE CAPTIONS

1. Correlation function for the objective analysis of altimeter data, as a function of distance. As an example, a 50km correlation radius has been chosen to draw this plot.
2. Average correlation function from a group of repeat tracks of SEASAT altimeter data in the SW Atlantic (from Fu, 1983).
3. Sea surface height variability in centimeters from GEOS-3 altimeter repeat pairs (from Douglas et al., 1983).
4. Sea surface height variability (contour interval = 1 cm) from repeat tracks of SEASAT altimeter data (from Cheney et al., 1983).
5. RMS expected error maps for a simulation of ERS-1 in a 1000 km square domain center on 29N,70W (POLYMODE Soviet current meter array). Maps are shown for days 1,2,3,4. Contour interval is 0.08. The parameters of the analysis are given in the text.
6. RMS expected error maps for a simulation of SPOT in a 1000 km square domain centered on 29N, 70W. Maps are shown for days 0,5,10,15. Contour interval is 0.07. Note westward propagation. The parameters of the analysis are given in the text.
7. Global coverage of ERS-1 (from Tavernier, 1983).
8. Global coverage of SPOT (from Tavernier, 1983).
9. Global coverage of TOPEX (from Tavernier, 1983).
10. Sea-truth, restored sea-surface height, RMS actual error field and RMS expected error field for a simulation of SPOT in a 281.25 km square domain centered on 29N, 70W (Harvard POLYMODE SDE Mark II data set). Contour intervals are: 2 cm, 2 cm, 1% of st. dev. of SSH, 5% of st. dev. of SSH. The westward propagation of the southern eddy and the stationarity of the northern one are reproduced.
11. Sea-truth, restored sea-truth (without satellite track), restored sea-truth (with satellite track), and results of 17 day 144 km baroclinic forecast using real data.
12. RMS expected error vs. cross-track distance for an objective analysis of 2 parallel tracks. The correlation radius was 50.0 km. Note the first minimum. The upward slope after the second minimum is an artifact of the integration in a finite domain.

## BIBLIOGRAPHY

- Carter, E.F., 1983: The statistics and dynamics of ocean eddies. Reports in Meteor. Oceanogr., 18, Harvard University.
- Cheney, R.E., J.G. Marsh and B.D. Beckley, 1983: Global mesoscale variability from collinear tracks of SEASAT altimeter data. J. Geophys. Res., 88(C7), 4343-4354.
- Douglas, B.C., R.E. Cheney and R.W. Agreen, 1983: Eddy energy of the northwest Atlantic and Gulf of Mexico determined from GEOS-3 altimetry. J. Geophys. Res., 88(C14), 9595-9603.
- Fu, L.L., 1983: On the wavenumber spectrum of oceanic mesoscale variability observed by the SEASAT altimeter. J. Geophys. Res., 88(C7), 4331-4342.
- Gandin, L.S., 1963: Objective Analysis of meteorological fields. Leningrad, Gidrometeorizdat, 266 pp.
- Menard, Y., 1983: Observation of eddy fields in the northwest Atlantic and northwest Pacific by SEASAT altimeter data. J. Geophys. Res., 88, 1853-1866.
- Miller R.N. and A.R. Robinson, 1983: Dynamical Forecast Experiments with a Baroclinic Quasigeostrophic Open Ocean Model, Proceedings of Conference on Predictability of Fluid Motion, La Jolla, Ca., G. Holloway, Editor (in press).
- Miller, R.N., A.R. Robinson and D.B. Haidvogel, 1983: A baroclinic quasigeostrophic open ocean model. J. Comp. Phys., 50, 38-70.
- Robinson, A.R., 1983: Description and prediction of oceanic fields: Data assimilation and optimal estimation. Proc. of COO/JOC (SCOR-WCRP) Study Conference on Large Scale Ocean Experiment in the World Climate Research Program, Tokyo, 1982.
- Robinson, A.R., J.A. Carton, C.N.K. Mooers, L.J. Walstad, E. F. Carter, M.M. Rienecker, J.A. Smith and W.G. Leslie, 1984: A real time dynamical forecast of ocean synoptic/mesoscale eddies, (submitted).
- Robinson, A.R., N.E. Huang, C.D. Leita0 and C.G. Parra, 1983: A study of the variability of ocean currents in the Northwestern Atlantic using satellite altimetry. J. Phys. Oceanogr., 13, 565-585.
- Robinson, A.R. and W.G. Leslie, 1984: Estimation and prediction of oceanic eddy fields, Prog. in Oceanogr., (In press).
- Tavernier, G., Projet POSEIDON: Echantillonnage spatio-temporel d'un satellite altimétrique pour la détermination des différentes composantes de la circulation océanique, ENSAE/GRGS, Toulouse, France, 1983.

Tu, K., 1981: A combined statistical and dynamical approach to regional forecast modelling of open ocean currents. Reports in Meteor. Oceanogr., 13, Harvard University.

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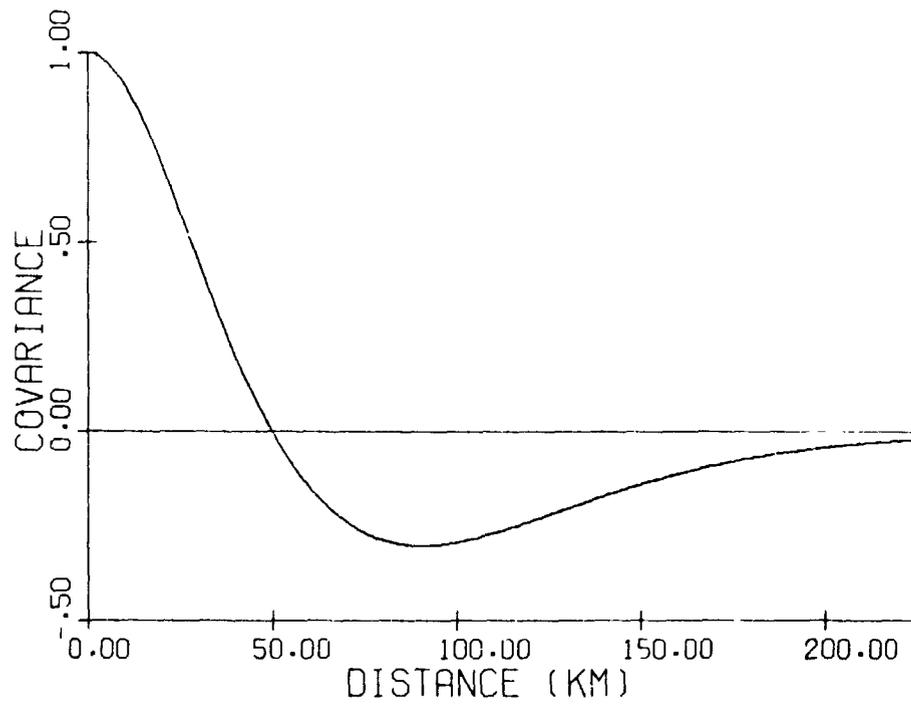


Figure 1

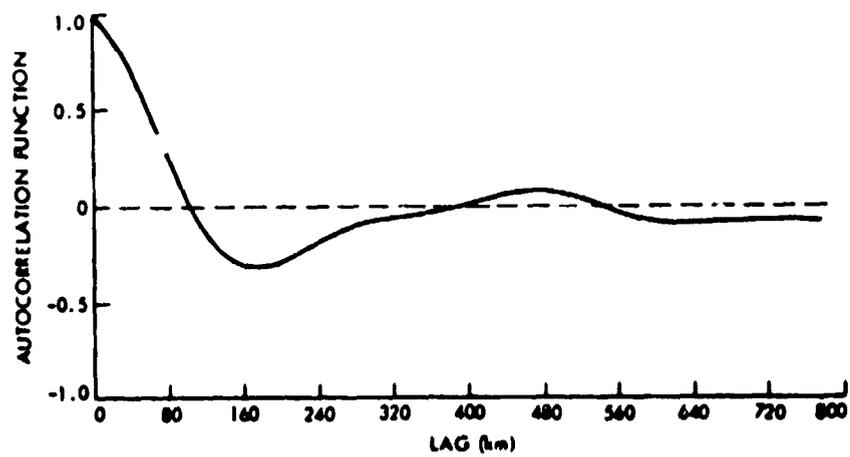


Figure 2

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Figure 3

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# SEASAT ALTIMETER MESOSCALE VARIABILITY

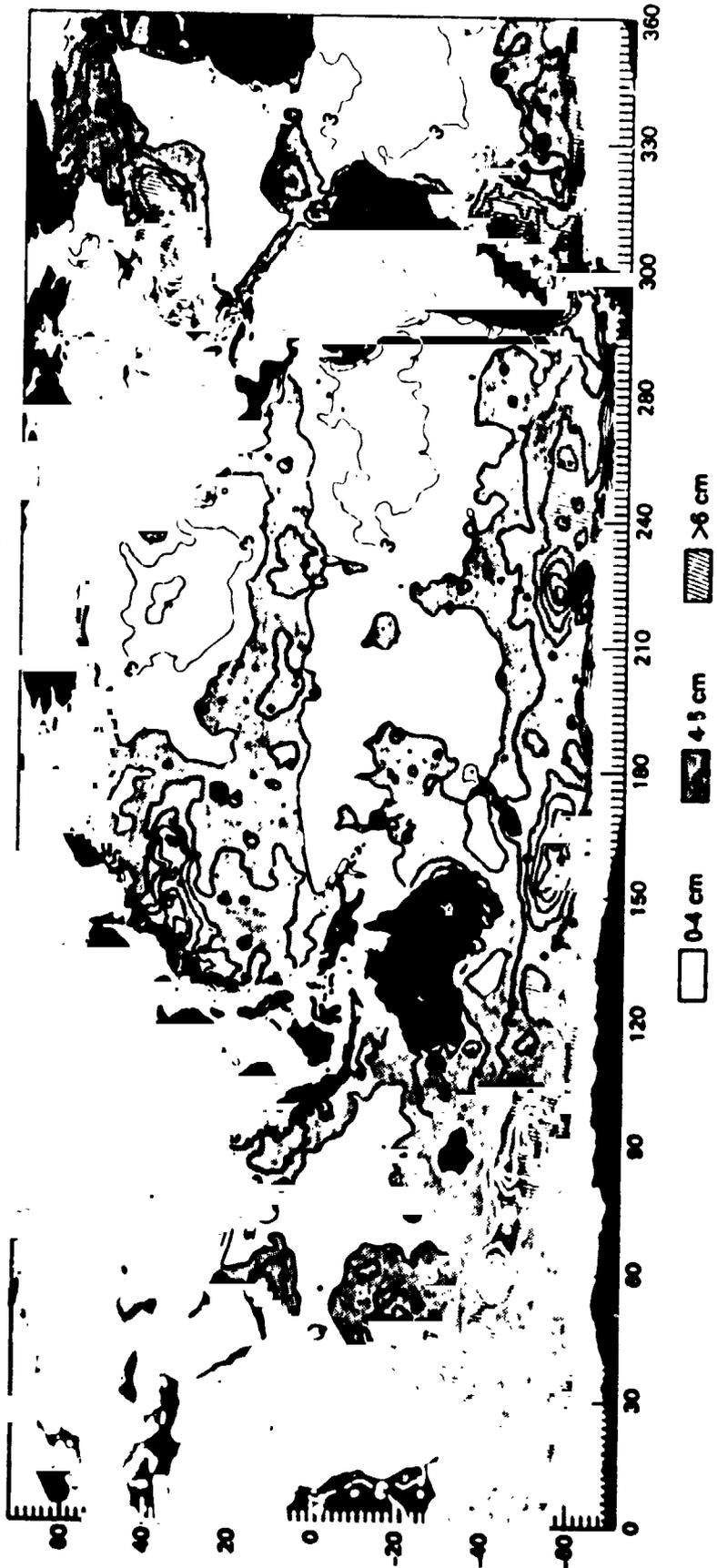


Figure 4

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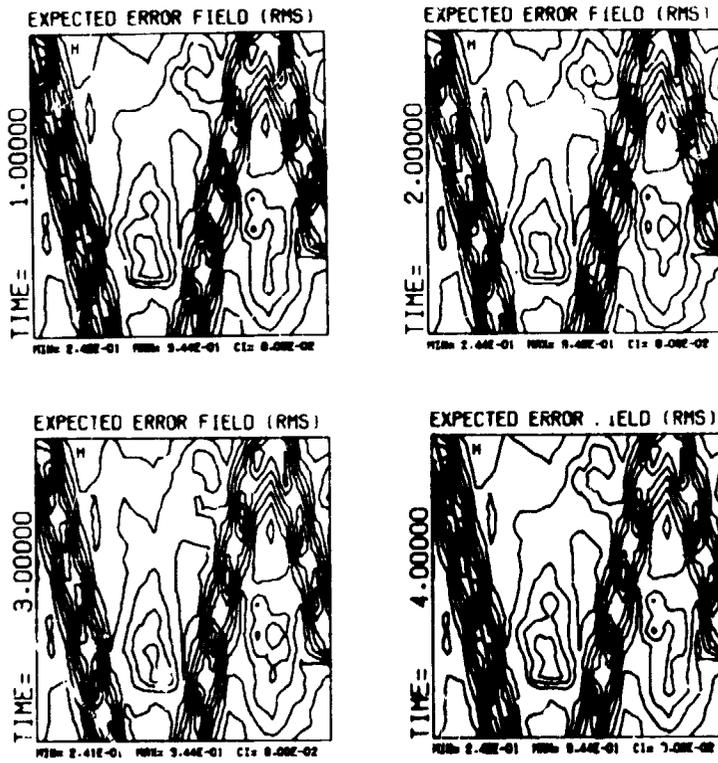


Figure 5 (ERS-1)

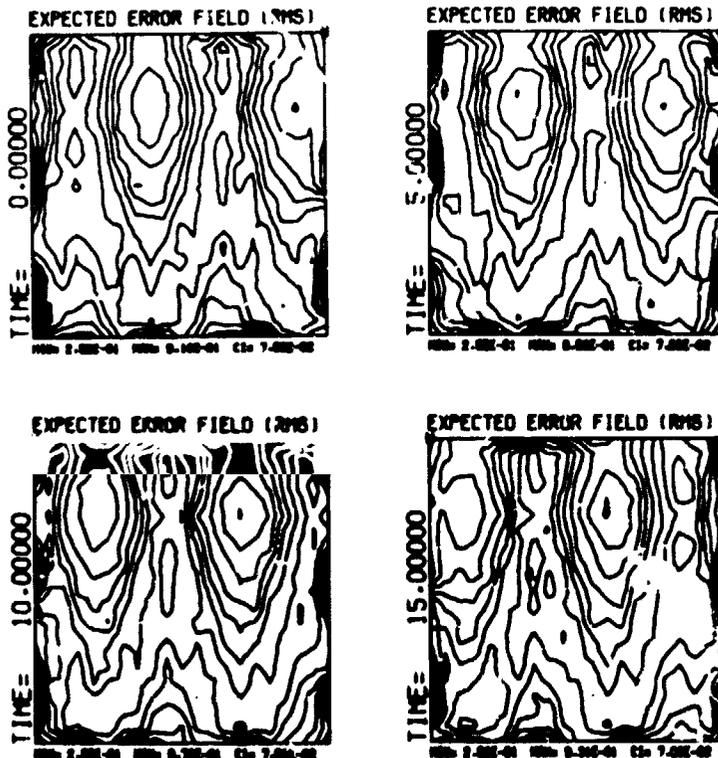


Figure 6 (SPOT)

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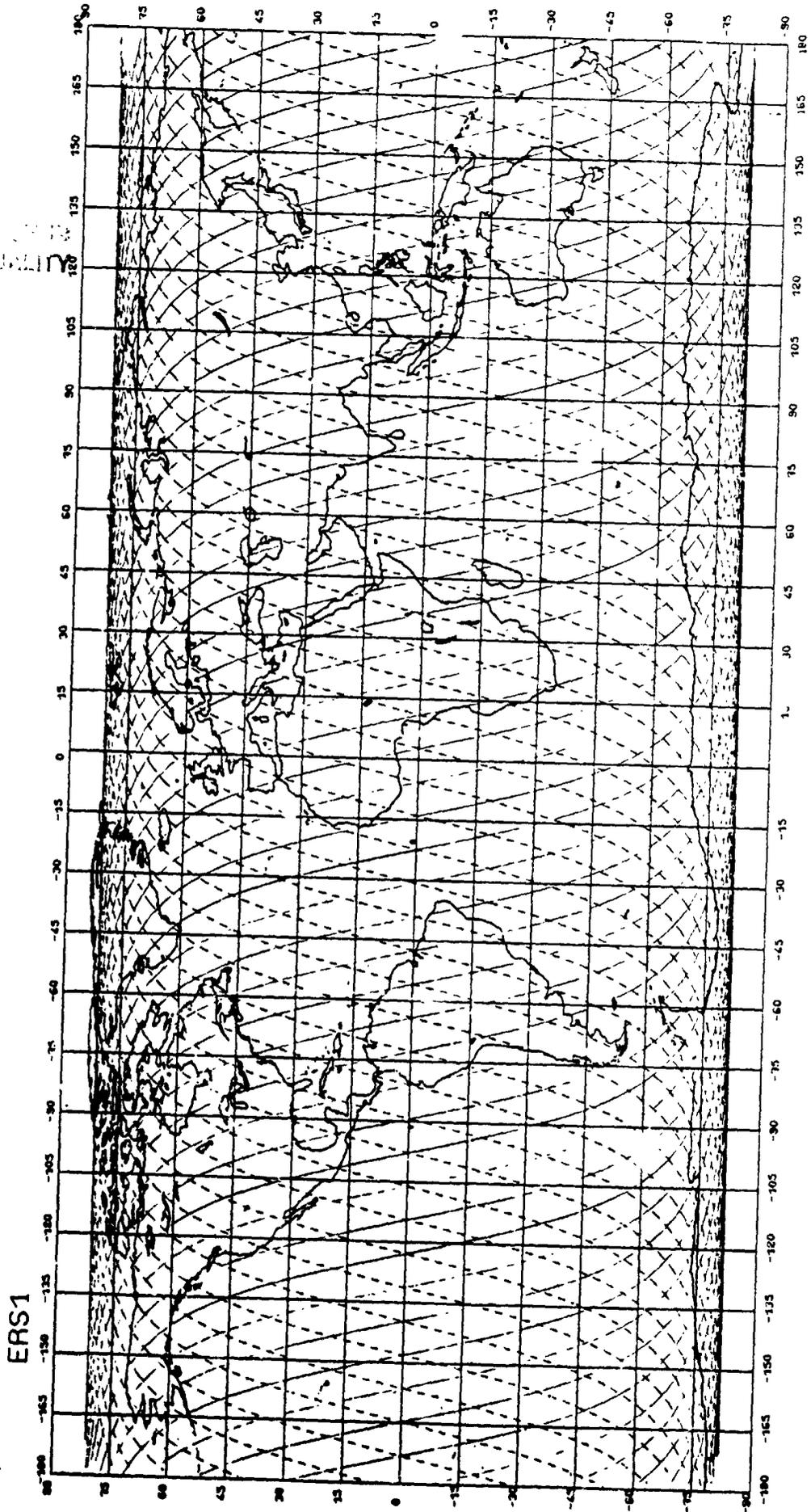
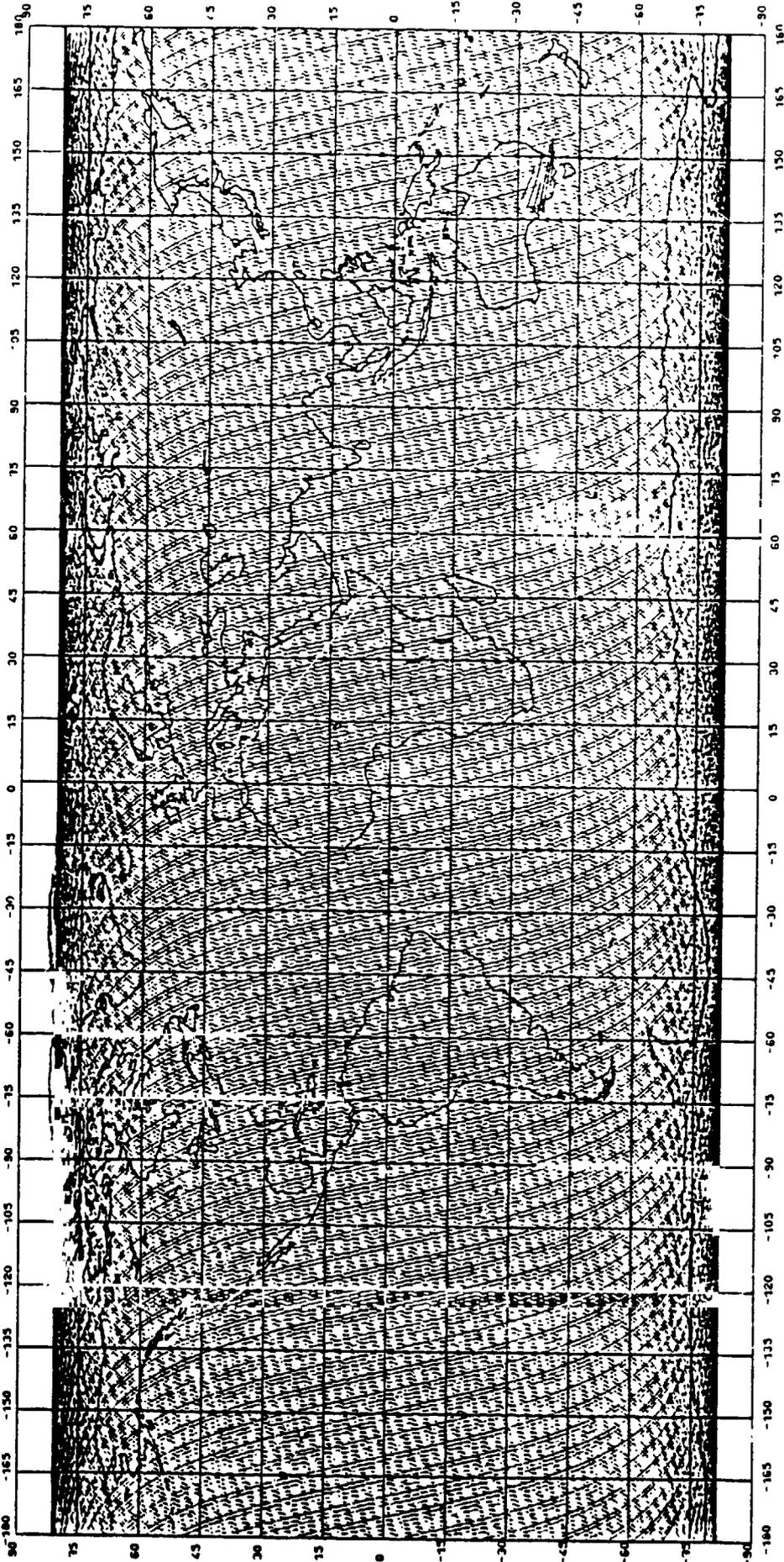


Figure 7

SPOT



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Figure 8

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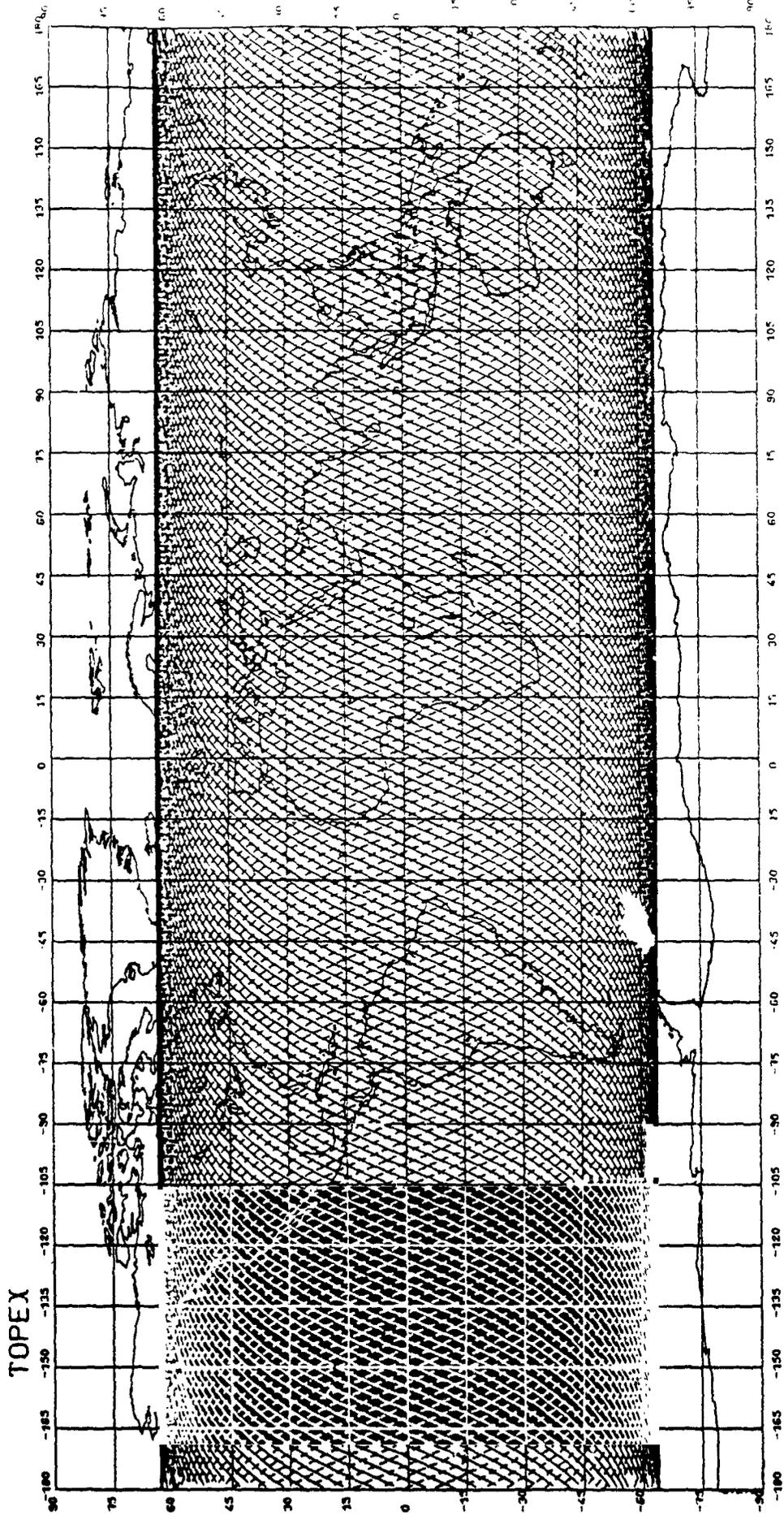


Figure 9

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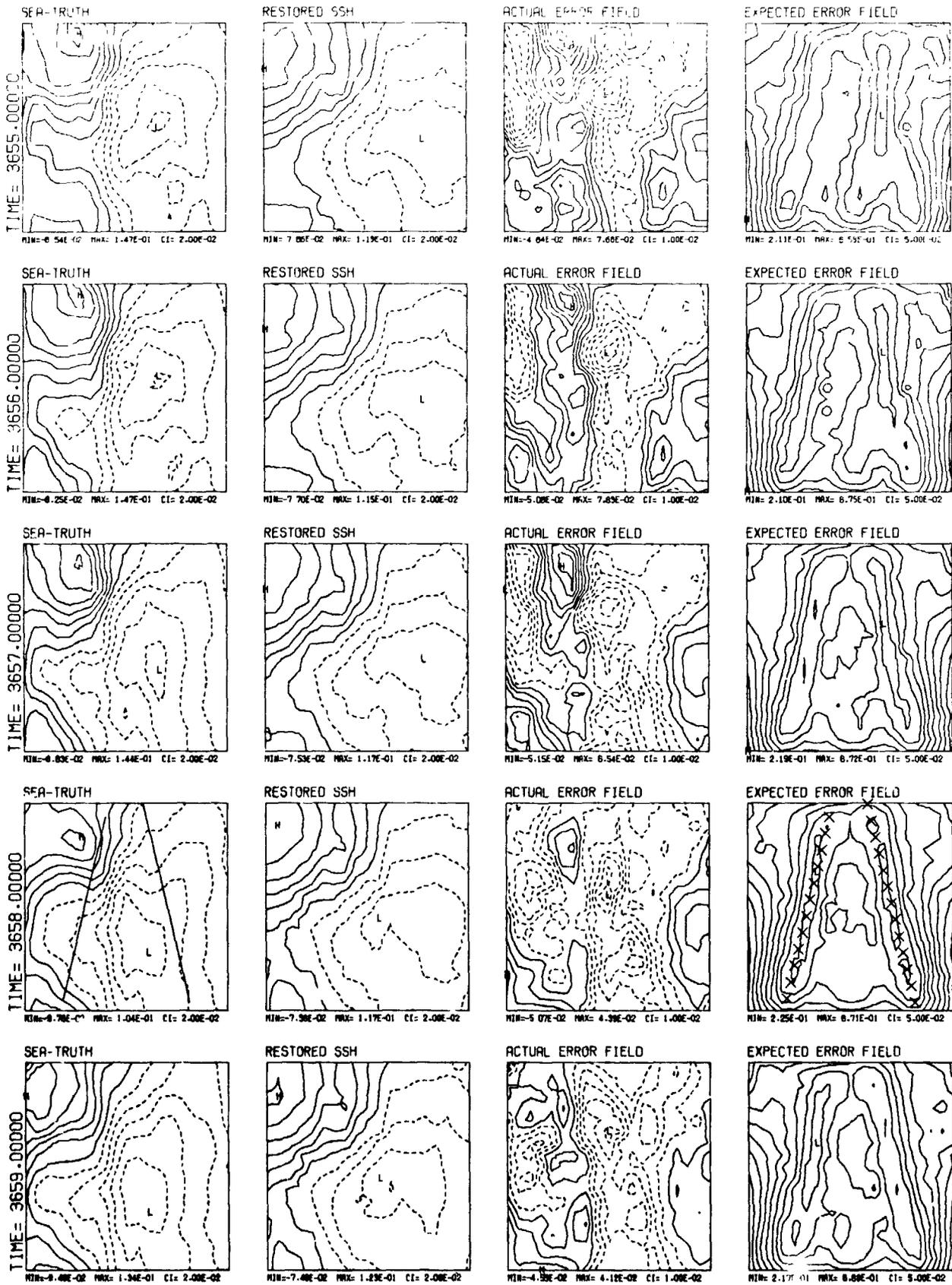
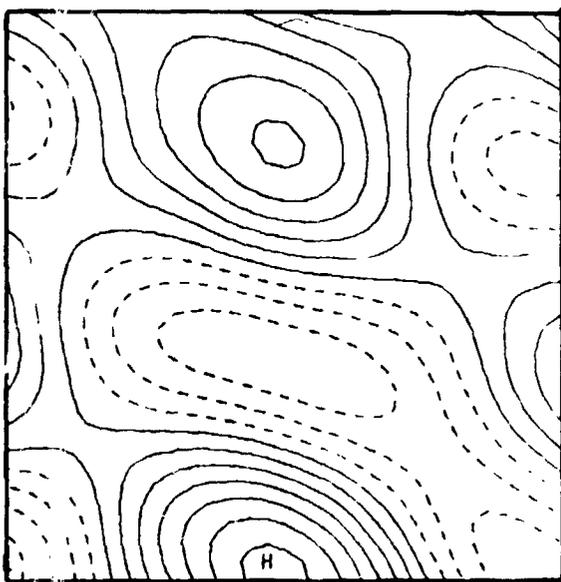
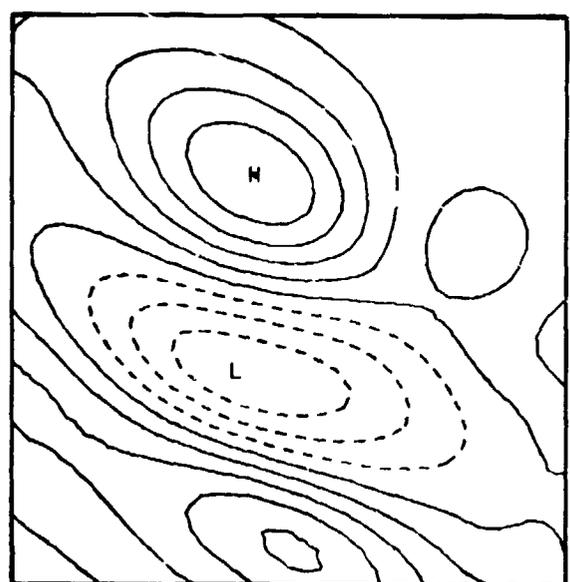


Figure 10

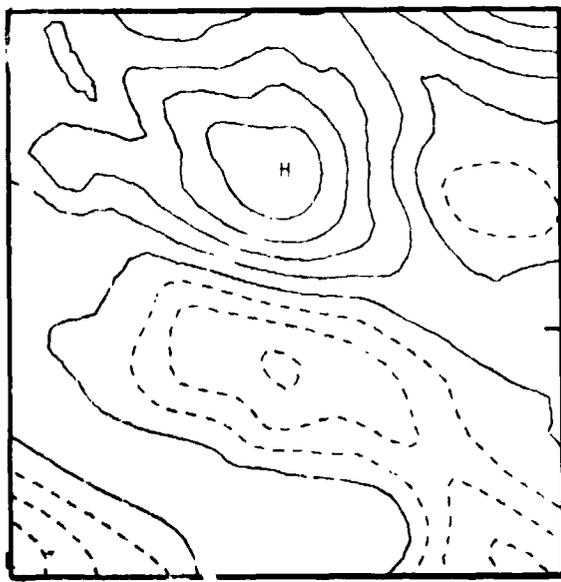
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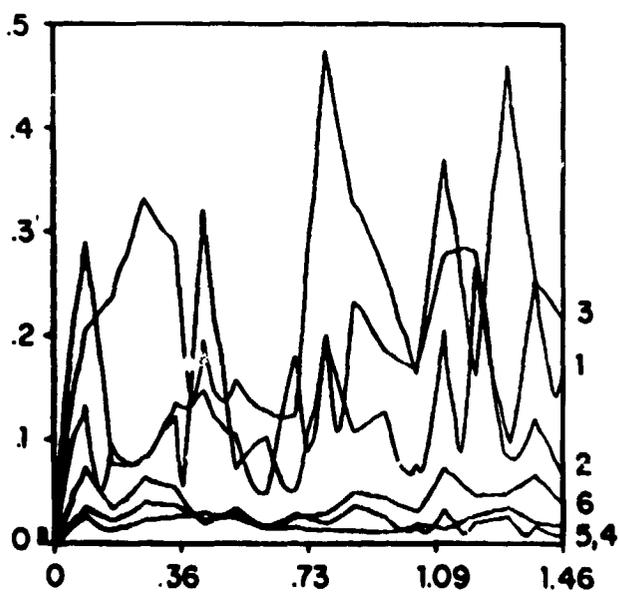
VERITAS PSI



OBJ ANAL PSI (without SAT)



OBJ ANAL PSI (with SAT)



17x17 BENCHMARK

RMS(DIFF(STFN))

Figure 11

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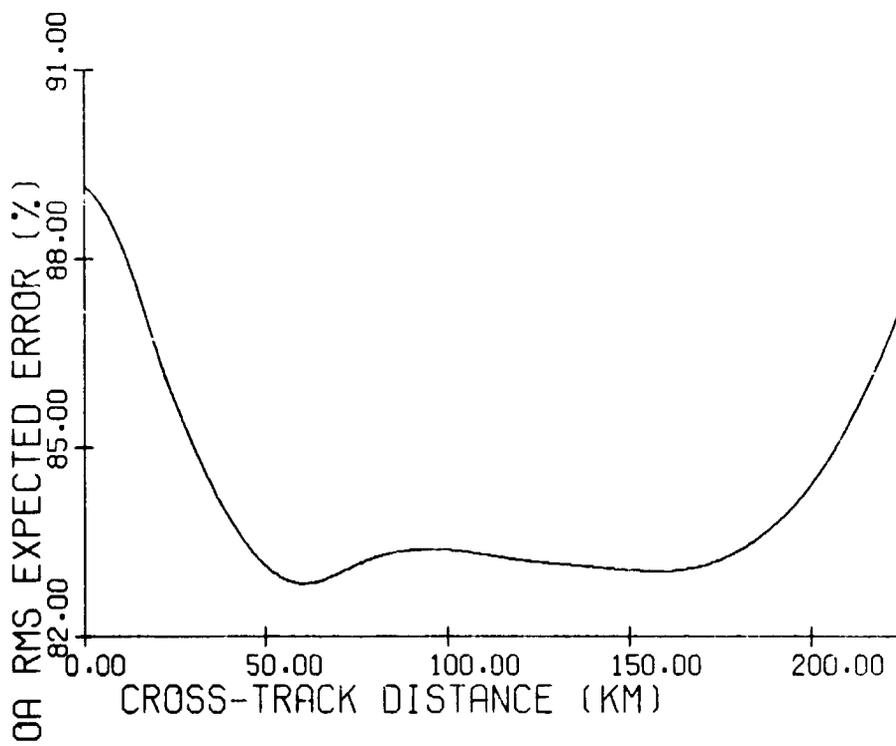


Figure 12